On Fuzzy Linear Programming Problems

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Abstract— In this paper, we concentrate on solving Fuzzy Linear Programming Problems (FLPP) in which the cost coefficients, the right-hand side vector and the technological coefficients are fuzzy numbers by defining a kind of fuzzy inequality between two fuzzy numbers and then compare the results obtained by solving fuzzy linear programming problems (FLPP) with the ranking function method of solving FLPP. In this paper we discuss the case of triangular fuzzy numbers.

Keywords: Fuzzy Set, Fuzzy Linear Programming Problem, Fuzzy Number, Fuzzy inequality.

I. INTRODUCTION

Linear Programming Problem (LPP) is a technique for the optimization of a linear objective function subject to a set of linear equality or inequality constraints. The feasible region of solutions of an LPP is a convex polyhedron. An LPP algorithm determines a point of the convex polyhedron where the objective function has its optimal value, if such a point exists. In 1947 George B. Dantzig invented the simplex method for solving such problems. However, in many real-life problems, data are not precise. It is often very difficult to satisfactorily solve the LPPs with the existing methods because the data is sometimes not absolute(crisp). In order to deal with such data LA Zadeh in 1965 introduced fuzzy sets and eventually fuzzy numbers. Fuzzy Linear Problems (FLPP) are those LPPs where either the cost coefficients or technological coefficients or the requirement values or the decision variables are fuzzy numbers. Bellman and Zadeh [11] first proposed the concept of decision-making problems in fuzzy environment. In this paper we introduce a method to solve a FLPP where the cost coefficients, technological coefficients and the requirement vectors are fuzzy numbers by defining an inequality between two fuzzy numbers.

In section II, we give some terminologies in fuzzy sets and in section III, results on FLPP are discussed with an example.

II. Preliminaries:

Definition: **1** (Fuzzy Set): Let X be a universal set. Then a fuzzy set $\tilde{A}$ in X is defined by its membership function $\mu_{\tilde{A}}: X \to [0,1]$ which assigns a real number denoted by $\mu_{\tilde{A}}(x)$ in the interval [0,1] to each element x of X, where $\mu_{\tilde{A}}(x)$ represents the degree of membership of x in $\tilde{A}$.

Definition: **2** (Support of a fuzzy set): The support of a fuzzy set $\tilde{A}$ in X is the crisp set of those points of X at which $\mu_{\tilde{A}}(x)$ is positive and is denoted by $Supp(\tilde{A})$. That is $Supp(\tilde{A}) = \{x \in X: \mu_{\tilde{A}}(x) > 0\} \subset X$.

Definition: **3** (Height of a fuzzy set): The height of a fuzzy set $\tilde{A}$ is denoted and defined by $Hgt(\tilde{A}) = Sup\{\mu_{\tilde{A}}(x): x \in X\}$.

Definition: 4 (Normal fuzzy set): A fuzzy set $\tilde{A}$ in X is called a normal fuzzy set if its height is unity, i.e., $\mu_{\tilde{A}}(x) = 1$ for some $x \in X$. If a fuzzy set is not normal it is called a subnormal fuzzy set.

Definition: **5** (Convex fuzzy set): A fuzzy set $\tilde{A}$ in $\mathbb{R}$ is said to be convex if $\forall x_1, x_2 \in \mathbb{R}$, and $\forall \lambda \in [0,1]$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$.

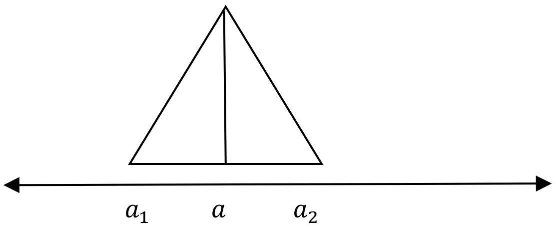
Definition: **6** (Fuzzy number): A fuzzy number is a fuzzy set on the real line $\mathbb{R}$ which is normal, convex and its membership function is piecewise continuous.

The class of all fuzzy numbers is denoted by $F(\mathbb{R})$.

A fuzzy number $\tilde{A}$ is called a positive fuzzy number if $\forall x < 0$, $\mu_{\tilde{A}}(x) = 0$ and a negative fuzzy number if $\forall x > 0$, $\mu_{\tilde{A}}(x) = 0$.

Definition: **7** (Triangular fuzzy number): A fuzzy number $\tilde{a}$ denoted by $\tilde{a} = (a_1, a, a_2)$ is called a triangular fuzzy number if its member function $\mu_{\tilde{a}}(x)$ is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, x < a_1 \\ \dfrac{x - a_1}{a - a_1}, a_1 \leq x \leq a \\ \dfrac{a_2 - x}{a_{2-a}}, a \leq x \leq a_2 \\ 0, x > a_2 \end{cases}$$



III. OPERATIONS, RANKING, DEFUZZIFICATION AND COMPARISION OF TRIANGULAR FUZZY NUMBERS

We define the addition, subtraction, multiplication and scalar multiplication as follows:

For $\tilde{a} = (a_1, a, a_2)$, $\tilde{b} = (b_1, b, b_2)$ and $k \in \mathbb{R}$, where $a_1 \geq 0, b_1 \geq 0$

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a + b, a_2 + b_2)$$

$$\tilde{a} - \tilde{b} = (a_1 - b_2, a - b, a_2 - b_1)$$

$$\tilde{a}\tilde{b} = (a_1 b_1, ab, a_2 b_2)$$

$$k\tilde{a} = \begin{cases} (ka_1, ka, ka_2), k \geq 0 \\ (ka_2, ka, 4a_1), k < 0 \end{cases}$$

If $\tilde{a} = (a_1, a, a_2)$, we define the ranking of $\tilde{a}$ by $R(\tilde{a}) = \frac{a_1 + 2a + a_2}{4}$.

Degree of fuzziness: For $\tilde{a} = (a_1, a, a_2)$. we define the degree of fuzziness of $\tilde{a}$ as follows: $DF(\tilde{a}) = a_2 - a_1$.

Proposition: Let $\tilde{a}$ be a triangular fuzzy number with degree of fuzziness D and $R(\tilde{a}) = R$. Then $\tilde{a}$ can be written in the form $\tilde{a} = (a, 3R - 2a - D, a + D)$, where $a \in \mathbb{R}$ such that $R - \frac{2D}{3} < a < R - \frac{D}{3}$.

For $\tilde{a} = (a_1, a, a_2)$ and $\tilde{b} = (b_1, b, b_2)$ we define $\tilde{a} = \tilde{b} \Leftrightarrow a_1 = b_1, a = b, a_2 = b_2$ and $\tilde{a} \leq \tilde{b} \Leftrightarrow b - b_1 \leq a - a_1, a \leq b, a_2 - a \leq b_2 - b$

IV. METHOD OF SOLVING FUZZY LPP

We consider the Fuzzy Linear Programming Problem (FLPP)

$max\ \tilde{Z} = \tilde{c}^T x$, subject to the constraints $\tilde{A}x \leq \tilde{b}$, $x \geq 0$, where $\tilde{c} = (\tilde{c}_1, \tilde{c_2} \cdots\cdots\cdots \tilde{c}_n)^T$, $x = (x_1, x_2, \cdots\cdots x_n)^T$, $\tilde{A} = (\tilde{a}_{ij})_{m \times n}$ and $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_m)^T$.

We now apply the operations of addition, subtraction and scalar multiplication between two triangular fuzzy numbers defined earlier in section III and finally applying the inequality between two fuzzy numbers, we convert the given FLPP into a crisp multi-objective LPP and then solve it by the simplex method.

Methodology: Given FLPP is $max\,\tilde{Z} = \tilde{c}^T x$, subject to the constraints $\tilde{A}x \leq \tilde{b}$, $x \geq 0$, where $\tilde{c} = (\tilde{c}_1, \tilde{c}_2 \cdots\cdots\cdots\cdots \tilde{c}_n)^T$, $x = (x_1, x_2, \cdots\cdots x_n)^T$, $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$ and $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_m)^T$, where c_j, a_{ij}, b_i are triangular fuzzy numbers.

Then, $max(Z_1, Z, Z_2) = (c_1^T x, c^T x, c_2^T x)$, subject to the constraints $(A_1x, Ax, A_2x) \leq (b_1, b, b_2)$, $x \geq 0$.

The above multi-objective LPP is then converted into three crisp LPPs as follows.

$min(Z - Z_1) = c^T x - c_1^T x$, subject to the constraints $Ax - A_1x \geq b - b_1$, $A_2x - Ax \leq b_2 - b$, $x \geq 0$.

$max\,Z = c^T x$, subject to the constraints $Ax \leq b$, $x \geq 0$.

$max(Z_2 - Z) = c_2^T x - c^T x$, subject to the constraints $Ax - A_1x \geq b - b_1$, $A_2x - Ax \leq b_2 - b$, $x \geq 0$.

These LPPs are then solved to find Z_1, Z and Z_2 and x from the second LPP. This finally gives $max\,\tilde{Z} = (maxZ - min\,Z_1\,, max\,Z\,, maxZ + max\,Z_2)$ and $x \geq 0$.

The process is explained with the help of following example.

Example: We consider the following problem with triangular fuzzy numbers, Same will be with trapezoidal fuzzy number which may be considered by the readers.

$max\,\tilde{Z} = (1,2,4)x_1 + (3,4,6)x_2$, subject to the constraints

$$(1,2,4)x_1 + (1,3,4)x_2 \leq (47,48,49)$$
$$(0,1,2)x_1 + (1,3,4)x_2 \leq (41,42,43)$$
$$(0,1,2)x_1 + (0,1,2)x_2 \leq (20,21,22)$$
$$x_1 \geq 0, x_2 \geq 0$$

Solution: Converting the given FLPP into three-objective crisp LPPs by using the operations of addition, and scalar multiplication and then applying the inequalities, we get

$max\,\tilde{Z} = (x_1 + 3x_2, 2x_1 + 4x_2, 4x_1 + 6x_2)$, subject to the constraints

$$(x_1 + x_2, 2x_1 + 3x_2, 4x_1 + 4x_2) \leq (47,48,49)$$

$$(0x_1 + x_2, x_1 + 3x_2, 2x_1 + 4x_2) \leq (41,42,43)$$
$$(0, x_1 + x_2, 2x_1 + 2x_2) \leq (20,21,22)$$
$$x_1, x_2, x_3 \geq 0$$

Now we form three crisp LPPs as follows:

$min\,Z_1 = (2x_1 + 4x_2) - (x_1 + 3x_2) = x_1 + x_2$, subject to the constraints

$$(2x_1 + 3x_2) - (x_1 + x_2) = x_1 + 2x_2 \geq 48 - 47 = 1$$
$$(4x_1 + 4x_2) - (2x_1 + 3x_2) = 2x_1 + x_2 \leq 49 - 48 = 1$$

$$(x_1 + 3x_2) - (0x_1 + x_2) = x_1 + 2x_2 \geq 42 - 41 = 1$$

$$(2x_1 + 4x_2) - (x_1 + 3x_2) = x_1 + x_2 \leq 43 - 42 = 1$$

$$x_1 + x_2 \geq 21 - 20 = 1$$

$$(2x_1 + 2x_2) - (x_1 + x_2) = x_1 + x_2 \leq 22 - 21 = 1$$

$$x_1, x_2 \geq 0$$

That is, $(A)\,min\,Z_1 = x_1 + x_2$, subject to the constraints

$x_1 + 2x_2 \geq 1, 2x_1 + x_2 \leq 1, x_1 + 2x_2 \geq 1, x_1 + x_2 \leq 1, x_1 + x_2 \geq 1 x_1 + x_2 \leq 1, x_1, x_2 \geq 0$

Similarly, $(B) max\, Z = 2x_1 + 4x_2$, subject to the constraints

$$2x_1 + 3x_2 \leq 48, x_1 + 3x_2 \leq 42, x_1 + x_2 \leq 21, x_1, x_2 \geq 0$$

And finally (C) $max\, Z_2 = 2x_1 + 2x_2$, subject to the constraints

$x_1 + 2x_2 \geq 1, 2x_1 + x_2 \leq 1, x_1 + 2x_2 \geq 1, x_1 + x_2 \leq 1, x_1 + x_2 \geq 1, x_1 + x_2 \leq 1. x_1, x_2 \geq 0$

Thus, the three LPPs are

$(A)\, min\, Z_1 = x_1 + x_2$, subject to the constraints

$$x_1 + 2x_2 \geq 1, 2x_1 + x_2 \leq 1, x_1 + x_2 \leq 1, x_1 + x_2 \geq 1, x_1, x_2 \geq 0$$

$(B) max\, Z = 2x_1 + 4x_2$, subject to the constraints

$$2x_1 + 3x_2 \leq 48, x_1 + 3x_2 \leq 42, x_1 + x_2 \leq 21, x_1, x_2 \geq 0$$

(C) $max\, Z_2 = 2x_1 + 2x_2$, subject to the constraints

$$x_1 + 2x_2 \geq 1, 2x_1 + x_2 \leq 1, x_1 + x_2 \leq 1, x_1 + x_2 \geq 1, x_1, x_2 \geq 0$$

The solutions of LPPS (A), (B) and (C) are as follows:

$(A)\, min\, Z_1 = 1$, $(B) max\, Z = 60, x_1 = 6, x_2 = 12$ and (C) $max\, Z_2 = 2$.

Hence, we have obtained the solution of the problem as $max\, \tilde{Z} = (60 - 1, 60, 60 + 2)$, i.e., $max\, \tilde{Z} = (59, 60, 62)$ and $x_1 = 6, x_2 = 12$. The value of the objective function after defuzzification is 60.25.

Now, we solve the given FLPP by the ranking function method.

The given FLPP after defuzzification by the ranking function defined above becomes a crisp LPP as follows:

$max\, Z = 2.25x_1 + 4.25x_2$, subject to the constraints

$$2.25x_1 + 2.75x_2 \leq 48$$
$$x_1 + 2.75x_2 \leq 42$$
$$x_1 + x_2 \leq 21$$
$$x_1, x_2 \geq 0$$

The solution of this LPP is $x_1 = 4.8, x_2 = 13.5273, max\, Z = 68.2909$.

V. CONCLUSION

I this work, we develop a new technique to solve Fuzzy Linear Programming problems by defining fuzzy inequality between two triangular fuzzy numbers. The method is then explained with an example. Then we compare the result obtained with the result obtained by the ranking function method. An analogous method could be applied for solving FLPP for trapezoidal fuzzy numbers.

REFERENCES

1. Reza Ghanbari et al, Fuzzy linear programming problems: models and solutions, Soft Computing, Vol. 24, 10043-10073, (2020).
2. Amit Kumar, Jagdeep Kaur, Pushpinder Singh, Fuzzy optimal solution of fully fuzzy linear programming problems with inequality constraints, International Journal of Mathematical and Computer Sciences, Vol. 6, 37-41, (2010).
3. Amit Kumar, Jagdeep Kaur, Pushpinder Singh, A new method for solving fully fuzzy linear programming problems, Applied Mathematical Modeling, Vol. 35, 817-823, (2011).

4. H. J. Zimmerman, Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Systems, Vol. 1, 45-55, (1978).
5. H. Tanaka, H. Ichihashi, K. Asai, Formulation of fuzzy linear programming problem based on comparison of fuzzy numbers, Control Cybernetics Vol. 3 (3), 185-194, (1991).
6. H. J. Zimmerman, Using fuzzy sets in operations research, EJOR, Vol. 13, 201-216, (1983)
7. H. J. Zimmerman, Optimization in fuzzy environment, Presented at XXI Int. TIMS and 46th ORSA conference, San Juan, Puerto Rico, (1974).
8. L. A. Zadeh, Fuzzy Sets, Information and Control, Vol. 8, 338-353, (1965).
9. Masatoshi Sakawa, Hitoshi Yano, Ichiro Nishizaki, Linear and multi-objective programming with fuzzy stochastic extensions, Springer.
10. Michael Gr. Voskoglou, Fuzzy Linear Programming Egyptian Computer Science Journal Vol. 42(4), 1-14, (2018).
11. R. E. Bellman, L. A. Zadeh, Decision Making in A Fuzzy Environment, Management Science, vol. 17, 141-164, (1970).