An Inventory Model with Partial Backlogging and Demand Depending on Circularity Index

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Abstract— In this paper we have created the inventory model which takes the Circularity Index and inflation into account. In this case, the circularity index determines demand. The circularity level determines the gross profit per unit. We take into account shortage in this partially backlogged model as well. The impact of carbon emissions is also considered, as is the need to invest in green technologies in order to reduce carbon emissions.

Key Words- Inflation, Partial Backlogging, circularity index.

I. INTRODUCTION AND LITERATURE REVIEW

The Economic Order Quantity (EOQ) model is a traditional inventory control model where all expenses are linear and demand is predictable and deterministic. After accounting for the product's circularity index, we revise the model's assumptions. To address global warming, biodiversity loss, carbon emissions, and waste, the circular economy has emerged as a viable alternative to industries' current linear economic framework. Every year, a massive amount of waste is dumped into the environment as a result of the current linear economic system, which begins with the production of goods from raw materials and ends with their disposal into the environment. Many raw materials are limited and require a significant amount of energy to extract. The circular economy is a systematic framework in which production is reused, recycled, remanufactured, and returned to the market at an economic cost. As a result, a circular economy addresses both environmental and financial issues. In Lenwandowski (2016) developed a comprehensive model to introduce the characteristics of a circular economy based on the business environment. In De Angelis (2018) developed a framework for circular supply chain to eliminate the drawbacks of the present linear supply chain. In Rabta(2020) for the first time, developed an economic order quantity model to investigate the results of products' circularity level. In Thomas and Mishra (2022) developed a sustainable circular economy model to reduce Carbon emission and waste with the help of 3D printing. Further, Khan et al.(2022) studied a production system with carbon emission to optimize the circular economy index policy.

Authors	**Circular economy**	**Demand depended on**	**Per unit gross profit depended on**	**Carbon emission**	**Shortages**	**Deterioration**
Chang *et al.* (2006)		Time	Constant			✓
Ouyang *et al.* (2009)		Constant	Constant			✓
Vandana *et al.* (2016)		Constant	Constant		Allowed with partial backlog	
Rabta (2020)	✓	Circularity index	Circularity index			
Thomas and Mishra (2022)	✓	Circularity index	Circularity index	✓		
Khan *et al.* (2023)	✓	Circularity index	Circularity index	✓		

This paper	✓	Circularity index	Circularity index	✓	Allowed with partial backlog	✓

Table:2.1

II. ASSUMPTIONS

Assumptions are made as follows.

1. The demand rate is dependent on circularity level

$$D(\omega) = D_o + alog(1 + \Upsilon\omega)$$

   Where $D_o = D(0)$, a, and Υ are constant parameters.

2. The unit gross profit is dependent on circularity level

$$p(\omega) = p_o + be^{\beta(\omega-1)}$$

   Where $p_o = p(0)$, b, and β are constant parameters.

3. This model contains a single item and single retailer.
4. Replenishment rate is instantaneous.
5. The inventory system has an infinite planning horizon.
6. Shortages are allowed with partial backlogging and the partial backlogging parameter is ρ, $0 < \rho < 1$.
7. Inflation is taken into consideration

III. NOTATIONS

A- Ordering cost per order
$p(\omega)$ -selling price per unit
p_o -selling price per unit when circularity index $\omega = 0$
$D(\omega)$ -demand rate

D_o -demand rate when circularity index $\omega = 0$

r -rate of inflation

C_e - reduce carbon emissions

Q -retailer's order quantity

θ -deterioration rate

B -back-order level

$I_1(t)$ -inventory level in interval $[0, T_1]$

$I_2(t)$ -inventory level in interval $[T_1, T]$

ρ -partial backlogging parameter

C_h -holding cost

C_d -deterioration cost

S_c -shortage cost

C_i -lost sale cost

IV. MATHEMATICAL MODEL FORMULATION

The behaviour of the inventory level is shown in figure 1.

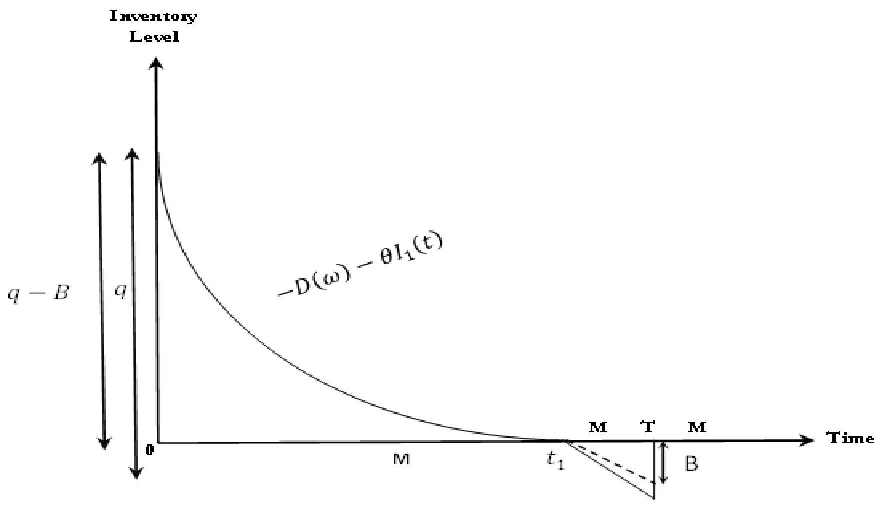


Fig:1

The differential equation representing the inventory level is:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -D(\omega), \qquad 0 \le t \le T_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -\rho D(\omega), \qquad T_1 \le t \le T \tag{2}$$

With boundary conditions $I_1(0) = q - B, I_1(T_1) = 0 = I_2(T_1), I_2(T) = -B$.

After solving these equations (1) and (2) with the help of boundary conditions, we get

$$I_1(t) = \frac{D(\omega)}{\theta}\left(e^{\theta T - t)} - 1\right), \qquad 0 \le t \le T_1 \tag{3}$$

$$I_2(t) = \rho D(\omega)(T_1 - t), \qquad T_1 \le t \le T \tag{4}$$

Ordered quantity and backorder for this model is

$$q = \frac{D(\omega)}{\theta}\{e^{\theta t_1} - 1 + \rho\theta(T - T_1)\}$$

and $$B = \rho D(\omega)(T - T_1).$$

The total profit for the retailer contains the following terms,

A. Ordering cost (OC) = A

B. Holding cost (C_h) = $C_h \int_0^{T_1} I_1(t)dt$

$$= \frac{C_h D(\omega)}{\theta}\left[\frac{1}{\theta + r}\left(e^{\theta T_1} - e^{-rT_1}\right) + \frac{1}{r}\left(e^{-rT_1} - 1\right)\right]$$

C. Deterioration cost (C_d)=$C_d\theta \int_0^{T_1} I_1(t)e^{-rt}dt$

$$= c_d D(\omega)\left[\frac{1}{\theta + r}\left(e^{\theta T_1} - e^{-rT_1}\right) + \frac{1}{r}\left(e^{-rT_1} - 1\right)\right]$$

D. Green investment = I_g

E. Carbon emission cost (CE) = $c_e \int_0^{T_1} e^{-rt} I_1(t)dt$

$$= \frac{C_e D(\omega)}{\theta}\left[\frac{1}{\theta + r}\left(e^{\theta T_1} - e^{-rT_1}\right) + \frac{1}{r}\left(e^{-rT_1} - 1\right)\right]$$

F. Shortage cost = $-S_c \int_{T_1}^{T} I_2(t) e^{-rt} dt$

$$= -\frac{S_c \rho D(\omega)}{r^2}\left[e^{-rT}(1 + rT - rT_1) - e^{(-rT_1)}\right]$$

G. Lost sale cost = $C_i \int_{T_1}^{T} (1-\beta) D(\omega) dt$

$$= \frac{C_i (1-\beta) D(\omega)}{r}\left[e^{-rT_1} - e^{-rT}\right]$$

H. Sales revenue = $p(\omega) \int_0^{T_1} e^{-rt} D(\omega) dt + p(\omega) B$

$$= \frac{p(\omega) D(\omega)}{r}\left[1 - e^{-rT_1}\right] + p(\omega) B$$

Total profit $= \left(\frac{p(\omega)D(\omega)}{r}\left[1 - e^{-rT_1}\right] + p(\omega)B\right) - \left(A + \frac{C_h D(\omega)}{\theta}\left[\frac{1}{\theta+r}\left(e^{\theta T} - e^{-rT_1}\right) + \frac{1}{r}\left(e^{-rT_1} - 1\right)\right] + c_d D(\omega)\left[\frac{1}{\theta+r}\left(e^{\theta T} - e^{-rT_1}\right) + \frac{1}{r}\left(e^{-rT_1} - 1\right)\right] + I_g + \frac{C_e D(\omega)}{\theta}\left[\frac{1}{\theta+r}\left(e^{\theta T_1} - e^{-rT_1}\right) + \frac{1}{r}\left(e^{-rT_1} - 1\right)\right] - \frac{S_c \rho D(\omega)}{r^2}\left[e^{-rT}(1 + rT - rT_1) - e^{(-rT_1)}\right] + \frac{C_i(1-\beta)D(\omega)}{r}\left[e^{-rT_1} - e^{-rT}\right]\right)$

V. NUMERICAL ILLUSTRATION

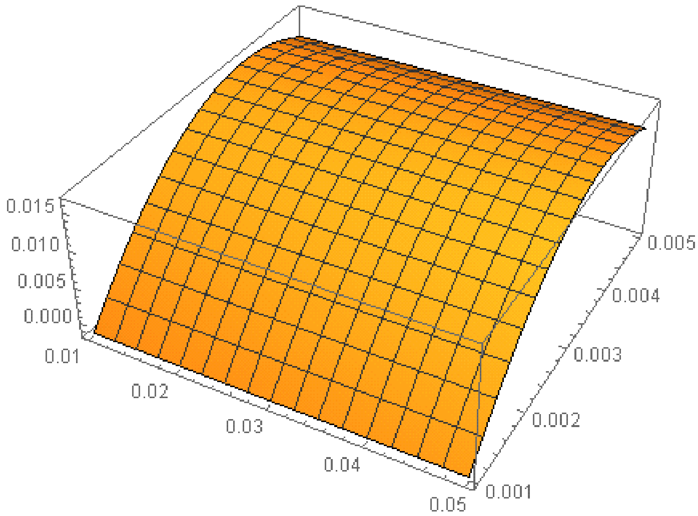
Parameters	Valu	Parameters	Values
p_0	1.5	b	0.1
β	0.21	r	0.05 %
D_o	1	A	0.01
γ	0.01	ρ	0.11
a	1.1	c_h	100$
θ	0.001	c_d	10$
c_e	310 Rs	s_c	0.001$

Final optimal solution is

T_1	Time taken to finish the inventory level $I_1(t)$	0.00381725 year
T	Replenishment cycle length	0.0927898 year
ω	Circularity index $0 \le \omega \le 1$	0.000707859
	Total profit	0.0160897 Lakh

VI. CONCAVITY

Fig: 2: concavity Between ω (Circularity index) and T_1(Time taken to finish the inventory level)



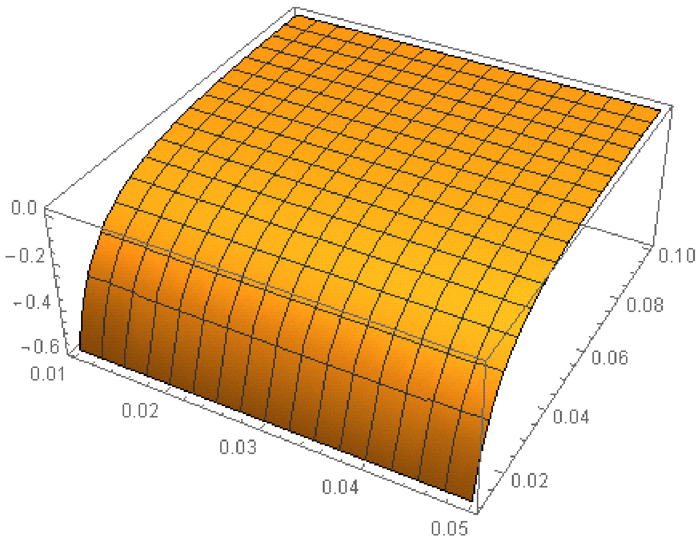


Fig:3: concavity Between ω (circularity index) and T (cycle length)

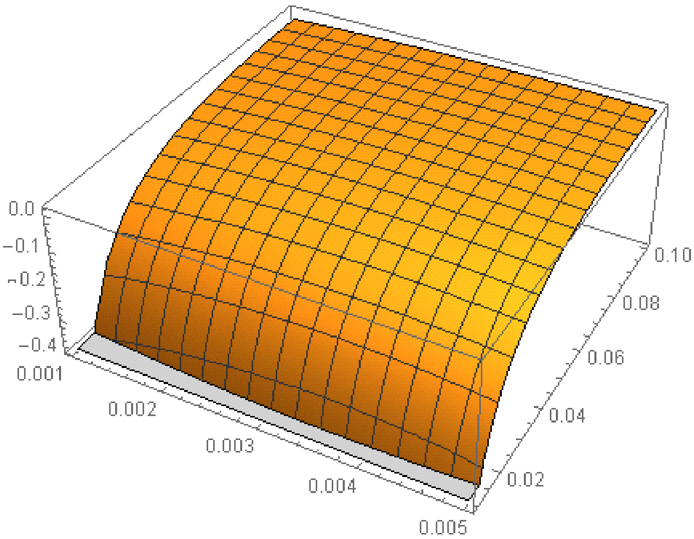


Fig:4: concavity Between T (cycle length) and T_1(Time taken to finish the inventory level)

VII. SENSITIVITY ANALYSIS

Parameter	% value	Total cost	T	T_1	ω
p_0	+20%	0.06138	0.00443	0.08645	0.00088
	+10%	0.03839	0.00412	0.08982	0.00050
	0	0.01648	0.00385	0.09266	0.00175
	-10%	-0.00560	0.003502	0.09542	0.00056
	-20%	-0.02673	0.00318	0.09769	0.000130
b	+20%	0.01853	0.00388	0.09317	0.01630
	+10%	0.01738	0.00387	0.09269	0.02830
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01490	0.00380	0.09293	0.000532
	-20%	0.013715	0.00378	0.09307	0.000468
β	+20%	0.01560	0.00381	0.09285	0.000503
	+10%	0.01584	0.003812	0.09282	0.0005011
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01634	0.003821	0.09277	0.000565
	-20%	0.0165	0.00382	0.09275	0.000557
r	+20%	0.016126	0.003817	0.09283	0.000960
	+10%	0.01610	0.00381	0.09281	0.000502
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01728	0.00383	0.09269	0.31492
	-20%	0.01728	0.00383	0.09269	0.31492

D_o	+20%	0.01853	0.00388	0.09317	0.01630
	+10%	0.01738	0.00387	0.09269	0.02830
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01490	0.00380	0.09293	0.000532
	-20%	0.013715	0.00378	0.09307	0.000468
a	+20%	0.06138	0.00443	0.08645	0.00088
	+10%	0.03839	0.00412	0.08982	0.00050
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01490	0.00380	0.09293	0.000532
	-20%	0.013715	0.00378	0.09307	0.000468
γ	+20%	0.06138	0.00443	0.08645	0.00088
	+10%	0.03839	0.00412	0.08982	0.00050
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01728	0.00383	0.09269	0.31492
	-20%	0.01728	0.00383	0.09269	0.31492
ρ	+20%	0.016126	0.003817	0.09283	0.000960
	+10%	0.01610	0.00381	0.09281	0.000502
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01490	0.00380	0.09293	0.000532
	-20%	0.013715	0.00378	0.09307	0.000468
A	+20%	0.016126	0.003817	0.09283	0.000960
	+10%	0.01610	0.00381	0.09281	0.000502
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01853	0.00388	0.09317	0.01630
	-20%	0.01738	0.00387	0.09269	0.02830
c_h	+20%	0.06138	0.00443	0.08645	0.00088
	+10%	0.03839	0.00412	0.08982	0.00050
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01728	0.00383	0.09269	0.31492
	-20%	0.01728	0.00383	0.09269	0.31492
θ	+20%	0.06138	0.00443	0.08645	0.00088
	+10%	0.03839	0.00412	0.08982	0.00050
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01490	0.00380	0.09293	0.000532
	-20%	0.013715	0.00378	0.09307	0.000468
c_d	+20%	0.016126	0.003817	0.09283	0.000960
	+10%	0.01610	0.00381	0.09281	0.000502
	0	0.01648	0.00385	0.09266	0.00175

	-10%	0.01728	0.00383	0.09269	0.31492
	-20%	0.01728	0.00383	0.09269	0.31492
c_e	+20%	0.06138	0.00443	0.08645	0.00088
	+10%	0.03839	0.00412	0.08982	0.00050
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01728	0.00383	0.09269	0.31492
	-20%	0.01728	0.00383	0.09269	0.31492
s_c	+20%	0.016126	0.003817	0.09283	0.000960
	+10%	0.01610	0.00381	0.09281	0.000502
	0	0.01648	0.00385	0.09266	0.00175
	-10%	0.01490	0.00380	0.09293	0.000532
	-20%	0.013715	0.00378	0.09307	0.000468

VIII. CONCLUSION

This paper is developed for single retailer. Numerical example is given to validate the model mathematically. Sensitivity analysis is carried out for showing the behaviour of different parameters on optimal solutions. On increases in p_0 the total cost, cycle length increases while circularity index decreases. On increases in the para meter b total cost, cycle length and circularity index increased. On increases in the parameter β total cost decreases cycle length fluctuating and circularity index decreases. On increases in the parameter r total cost, cycle length and circularity index decreases. On increases in the parameter D_0 total cost, cycle length and circularity index increases. On increases in the parameter a total cost, cycle length increases while circularity index fluctuating. On increases in the parameter ρ the total cost, cycle length and circularity index increases.

REFERENCES

1. De Angelis, R., Howard, M., & Miemczyk, J. (2018). Supply chain management and the circular economy: towards the circular supply chain. *Production Planning & Control*, *29*(6), 425-437.
2. Dye, C. Y., Chang, H. J., & Teng, J. T. (2006). A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. *European Journal of Operational Research*, *172*(2), 417-429.
3. Khan, M. A. A., Shaikh, A. A., Cárdenas-Barrón, L. E., Mashud, A. H. M., Treviño-Garza, G., & Céspedes-Mota, A. (2022). An inventory model for non-instantaneously deteriorating items with nonlinear stock-dependent demand, hybrid payment scheme and partially backlogged shortages. *Mathematics*, *10*(3), 434.
4. Lewandowski, M. (2016). Designing the business models for circular economy—Towards the conceptual framework. *Sustainability*, *8*(1), 43.
5. Ouyang, L. Y., Ho, C. H., & Su, C. H. (2009). An optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit. *Computers & Industrial Engineering*, *57*(3), 920-930.
6. Rabta, B. (2020). An Economic Order Quantity inventory model for a product with a circular economy indicator. *Computers & Industrial Engineering*, *140*, 106215.
7. Thomas, A., & Mishra, U. (2022). A sustainable circular economic supply chain system with waste minimization using 3D printing and emissions reduction in plastic reforming industry. *Journal of Cleaner Production*, *345*, 131128.
8. Vandana, & Srivastava, H. M. (2017). An inventory model for ameliorating/deteriorating items with trapezoidal demand and complete backlogging under inflation and time discounting. *Mathematical Methods in the Applied Sciences*, *40*(8), 2980-2993.